Field Junior School Calculation Policy

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new National Curriculum commencing September 2014. Through the policy, we aim to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. **At Field Junior School, we intend that the manipulatives outlined in the policy be used as a scaffold to allow our children to understand the process and enable them to move on to the abstract form.** We know that school wide policies, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to 9 + 9 and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

Progression in addition and subtraction

Addition and subtraction are connected.

Part	Part
Whole	

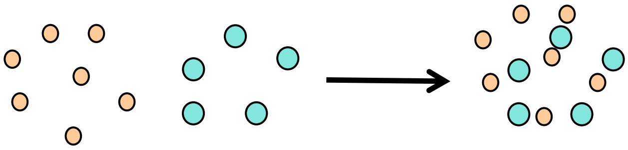
Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

Addition- EYFS/Year 1

Combining two sets (aggregation)

Putting together – two or more amounts or numbers are put together to make a total

7 + 5 = 12



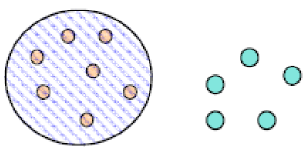
Count one set, then the other set. Combine the sets and count again. Counting starts at 1.

Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting.

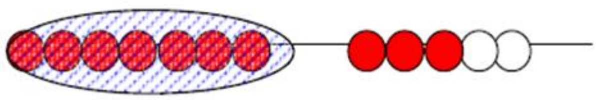
Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

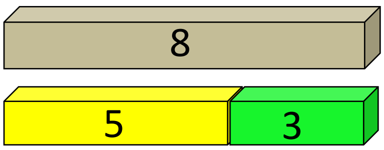
Bead strings:



Make a set of 7 and a set of 5. Then count on from 7.

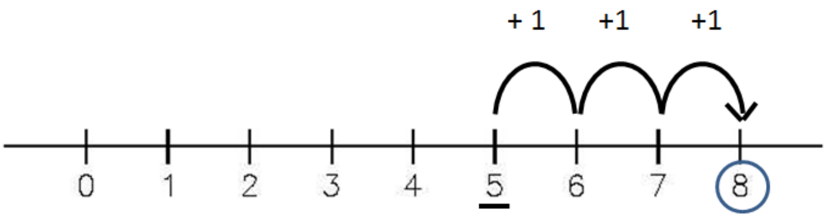
Cuisenaire Rods:

5 + 3 = 8



Start with a 5 rod. Add a 3 rod. What rod does the total equal?

Number line:



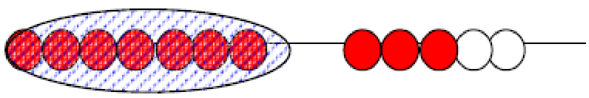
Start on 5 then count on 3 more.

Addition- Year 1

Bridging through 10s

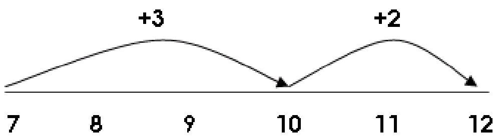
This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



7 + 5 is decomposed / partitioned into 7 + 3 + 2.
The bead string illustrates ‘how many more to the next multiple of 10?’ (link to number bonds)
Then ‘if we have used 3 of the 5 to get to 10, how many more do we need to add on?

Number line



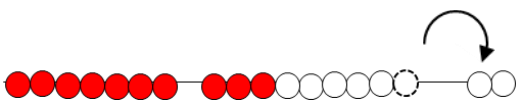
How many to the nearest ten? How many more do I need to add?

Compensation model (adding 9 and 11)

This model of calculation encourages efficiency and application of known facts (how to add ten)

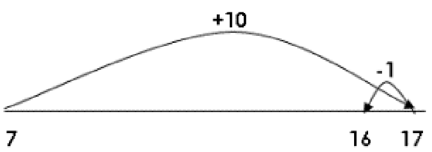
7 + 9

Bead string:



Children find 7, then add on 10 and then adjust by removing 1.

Number line:



Herts for Learning

Addition- Year 2

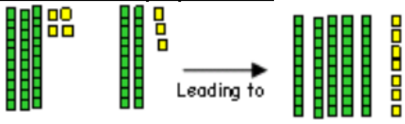
Working with larger numbers
Tens and units + tens and units

Ensure that the children are familiar with Base 10 equipment

Partitioning (Aggregation model)

34 + 23 = 57

Base 10 equipment:

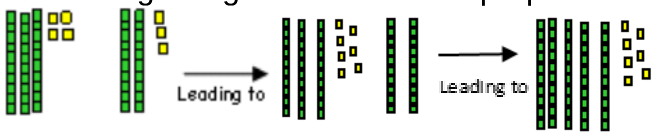


Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Base 10 equipment:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



Number line:

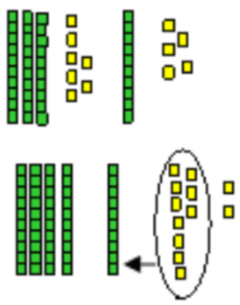
Children start on the greater number and add tens in one step and units in another.

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Using the Base 10 equipment, children exchange ten units for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

37 + 15 = 52



Always start with larger number. Count on from larger number.
Ten units are exchanged for a tens rod.

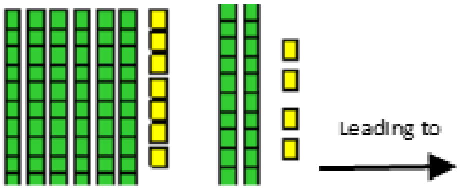
Addition- Year 3

Expanded Vertical Method (up to 3 digits)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.

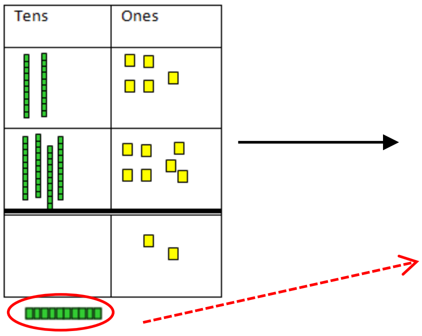
Base 10 equipment:

67 + 24 = 91



$$
\begin{array}{rrl}
+ & 67 = & 60 + 7 \\
 & 24 = & \underline{20 + 4} \\
 & & \underline{80 + 11} = 91
\end{array}
$$

Compact method

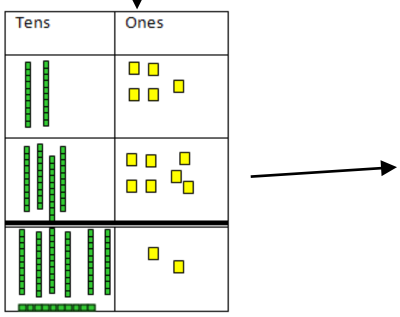


$$
\begin{array}{rcc}
 & T & U \\
+ & 4 & 7 \\
 & \underline{2} & \underline{5} \\
 & & \underline{2} \\
 & {\scriptstyle 1} &
\end{array}
$$

(A ten exchanged under)

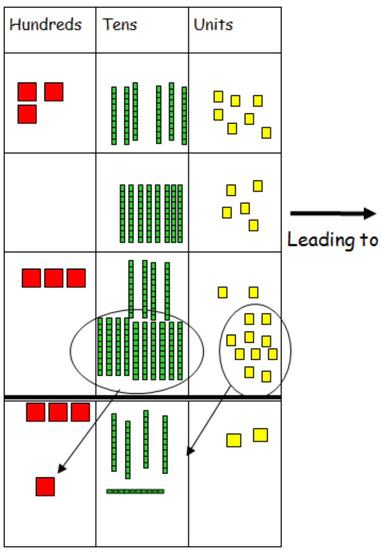
Add the units together, exchanging 10 units for a tens rod if necessary.

Leading to



$$
\begin{array}{rcc}
 & T & U \\
+ & 4 & 7 \\
 & \underline{2} & \underline{5} \\
 & \underline{7} & \underline{2} \\
 & {\scriptstyle \cancel{1}} &
\end{array}
$$

Add the tens together, including the exchanged one. Remember to cross out the exchanged numbers once used.



$$
\begin{array}{rccc}
 & H & T & U \\
+ & 3 & 6 & 7 \\
 & & \underline{8} & \underline{5} \\
 & \underline{4} & \underline{5} & \underline{2} \\
 & {\scriptstyle \cancel{1}} & {\scriptstyle \cancel{1}} &
\end{array}
$$

Extend on two previous steps by exchanging ten x 10 rods for a one hundred flat block.

Addition- Year 4

Year 4 will use the same expanded vertical and compact methods as Year 3; however, in Year 4, the method will continue with increasingly larger numbers up to four digits. By the end of Year 4, children must be secure in abstract written column method up to four digits without use of manipulatives.

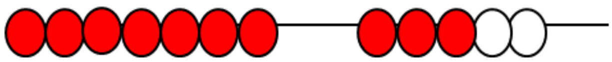
Vertical acceleration

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals (for money and measure)

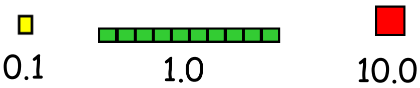
Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Base 10 equipment:



Addition of decimals

Aggregation model of addition

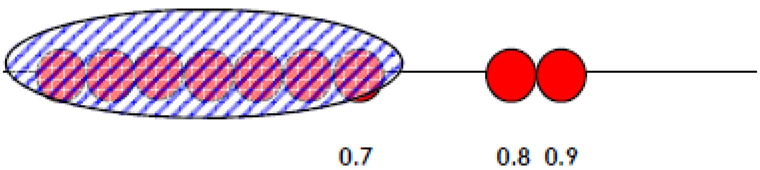
Counting both sets – starting at zero.

0.7 + 0.2 = 0.9

Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

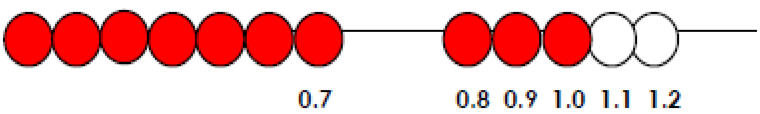
0.7 + 0.2 = 0.9



Bridging through 1.0

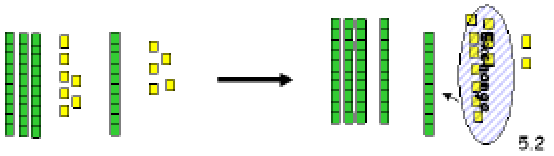
Encouraging connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning

3.7 + 1.5 = 5.2



Addition- Year 5

In Year 5, decimal addition should then be transferred across into compact column method in the same way as it was with whole numbers in Year 4.
By the end of Year 5, all children must be secure in abstract written column method upto six digits or 2 decimal places without use of manipulatives.

Gradation of difficulty- addition:

Lower School

1. No exchange
e.g. 7 + 2 = 9
14 + 13 = 27

$$\begin{array}{ccc} & T & U \\ + & 1 & 4 \\ & 1 & 3 \\ \hline & 2 & 7 \\ \hline \end{array}$$

2. Exchanging ones to tens
e.g. 14 + 9 = 23
127 + 45 = 172

$$\begin{array}{cccc} & H & T & U \\ + & 1 & 2 & 7 \\ & & 4 & 5 \\ \hline & 1 & 7 & 2 \\ \hline & & \cancel{1} & \end{array}$$

7 + 5 in units column = 12
1 from <u>1</u>2 is exchanged for a ten

3. Exchanging tens to hundreds
e.g. 82 + 42 = 124
261 + 175 = 436

$$\begin{array}{cccc} & H & T & U \\ + & 2 & 6 & 1 \\ & 1 & 7 & 5 \\ \hline & 4 & 3 & 6 \\ \hline & \cancel{1} & & \end{array}$$

6 + 7 in tens column = 13
1 from <u>1</u>3 is exchanged for a hundred

4. Exchanging ones to tens and tens to hundreds
e.g. 157 + 289 = 446
148 + 373 = 521

$$\begin{array}{cccc} & H & T & U \\ + & 3 & 7 & 3 \\ & 1 & 4 & 8 \\ \hline & 5 & 2 & 1 \\ \hline & \cancel{1} & \cancel{1} & \end{array}$$

8 + 3 in units column = 11
1 from <u>1</u>1 is exchanged for a ten
<u>AND</u>
7 + 4 + 1 in tens column = 12
1 from <u>1</u>2 is exchanged for a hundred

5. More than two numbers in calculation
e.g. 12 + 17 + 34 = 63
162 + 241 + 155 = 558

$$\begin{array}{cccc} & H & T & U \\ + & 2 & 4 & 1 \\ & 1 & 6 & 2 \\ & 1 & 5 & 5 \\ \hline & 5 & 5 & 8 \\ \hline & \cancel{1} & & \end{array}$$

6. As 5 but with different number of digits
e.g. 152 + 73 + 169 = 394
8 + 372 + 54 = 434

$$\begin{array}{cccc} & H & T & U \\ + & 3 & 7 & 2 \\ & & 5 & 4 \\ & & & 8 \\ \hline & 4 & 3 & 4 \\ \hline & \cancel{1} & \cancel{1} & \end{array}$$

Gradation of difficulty- addition:

Upper School

7. Decimals up to 2 decimal places (same number of decimal places)
e.g. 2.16 + 4.53 = 6.69
1.34 + 2.72 = 4.06

$$\begin{array}{cc} & \text{U . t h} \\ + & 2.72 \\ & 1.34 \\ \hline & 4.06 \\ \hline & \cancel{1} \end{array}$$

8. Add two or more decimals with a range of decimal places
e.g. 1.9 + 2.33 + 8.6 = 12.83
2.4 + 12 + 5.73 = 20.13

$$\begin{array}{cc} & \text{T U . t h} \\ + & 12.00 \\ & 05.73 \\ & 02.40 \\ \hline & 20.13 \\ \hline & \cancel{1}\ \cancel{1} \end{array}$$

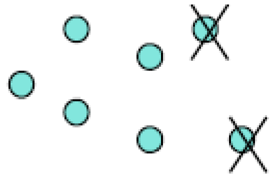
Gaps will be filled with a 0 place holder.

Subtraction- EYFS/Year 1

Taking away (separation model)

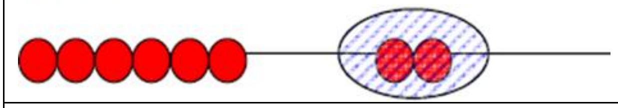
Where one quantity is taken away from another to calculate what is left.

7 – 2 = 5



Bead strings:

8 – 2 = 6

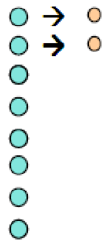


Finding the difference (comparison model)

Two quantities are compared to find the difference.

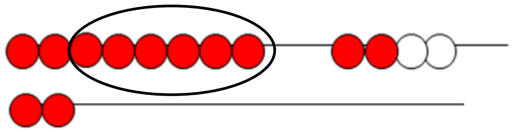
8 – 2 = 6

Counters:



Count the two sets of counters. Count how many more one set has/the difference.

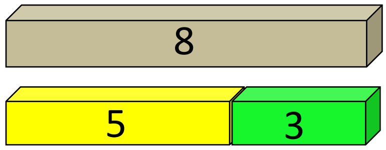
Bead strings:



Line up a set of 8 next to a set of 2. Then count the difference.

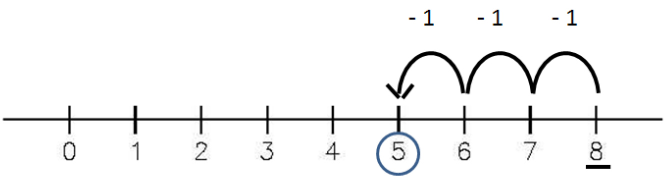
Cuisenaire Rods:

8- 5 = 3



Line up the 8 and the 5. What size rod do I need to fill the gap?

Number line:



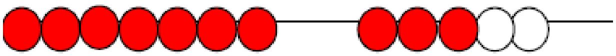
Start on 8 then count back 5.

Subtraction- Year 1

Bridging through 10s

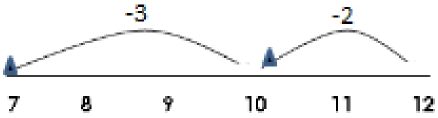
This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



12 – 7 is decomposed / partitioned in 12 – 2 – 5.
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back?

Number Line:



Count back to the nearest 10. How many more do I need to count back?

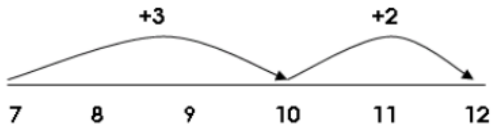
Counting up or 'Shop keepers' method

Bead string:

12 – 7 becomes 7 + 3 + 2.

Starting with 7, how many more do I need to get to 12?
Starting from 7 on the bead string 'how many more to the next multiple of 10?' (link to number bonds), 'how many more to get to 12?'.

Number Line:



Count up to nearest 10. How many more do I need to count on?

Compensation model (adding 9 and 11)

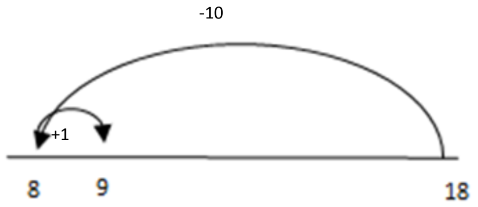
This model of calculation encourages efficiency and application of known facts (how to add ten)

18 – 9

Bead string:

Children find 18, then subtract 10 and then adjust by adding 1.

Number line:



Subtract 10 and add one.

Subtraction- Year 2

Working with larger numbers

Tens and units + tens and units

Ensure that the children are familiar with Base 10 equipment

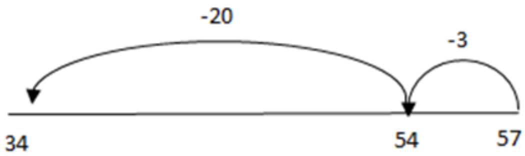
Take away (Separation model)

57 – 23 = 34

Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the units and then the tens. In preparation for formal decomposition.

Number Line:



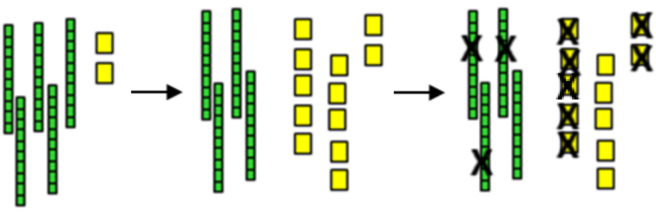
Subtract the units and then the tens.

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Using the Base 10 equipment, children exchange ten units for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

52 – 37 = 15



Count out 52 Base 10. Remove 37, exchanging one tens rod for ten units.

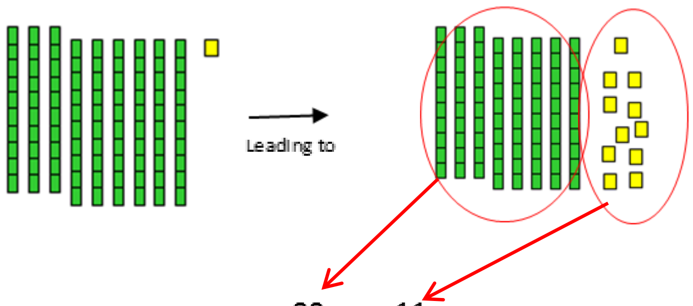
Subtraction- Year 3

Expanded Vertical Method (up to 3 digits)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.

Base 10 equipment:

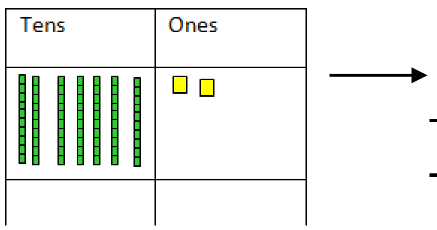
91 – 67 = 24



$$\begin{array}{rrcr} & 80 & & 11 \\ & \cancel{90} & + & \cancel{1} \\ - & 60 & + & 7 \\ \hline & 20 & + & 4 \end{array}$$

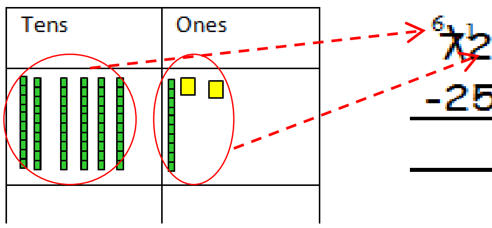
Partition 91 into 90 + 1. Partition 67 into 60 + 7. Working vertically, take 7 from 1, which cannot be done so exchange a 10 from your 90. 90 + 1 becomes 80 + 11. Then complete 11 – 7 and 80 – 60.

Compact decomposition



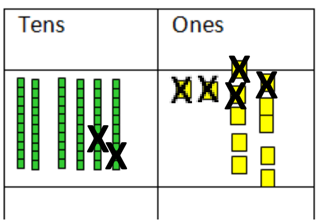
$$\begin{array}{r} 72 \\ -25 \\ \hline 47 \\ \hline \end{array}$$

Count out 72 Base 10 blocks into the place value grid



$$\begin{array}{r} {}^{6}\cancel{7}{}^{1}2 \\ -25 \\ \hline \\ \hline \end{array}$$

Subtract your units first, exchanging from tens if necessary



$$\begin{array}{r} {}^{6}\cancel{7}{}^{1}2 \\ -25 \\ \hline 47 \\ \hline \end{array}$$

Subtract 5 units and 2 tens, recording how many units and tens remain vertically

Subtraction- Year 4

Year 4 will use the same expanded vertical and compact methods as Year 3; however, in Year 4, the method will continue with increasingly larger numbers up to four digits. By the end of Year 4, children must be secure in abstract written column method up to four digits without use of manipulatives.

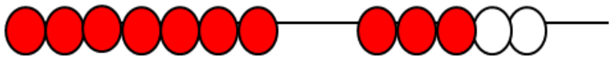
Vertical acceleration

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals (for money and measure)

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

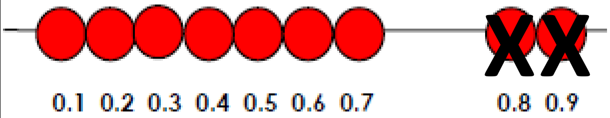
Base 10 equipment:

Subtraction of decimals

Take away model

0.9 – 0.2 = 0.7

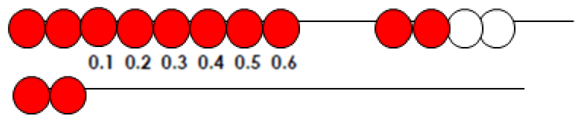
Count out 0.9. Remove 0.2. Count what is left.



Finding the difference (or comparison model):

0.8 – 0.2 = 0.6

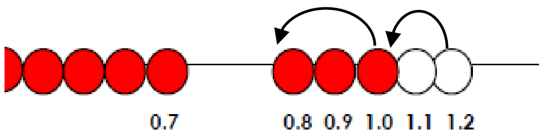
Line up 0.8. Line up 0.2. Count the difference.



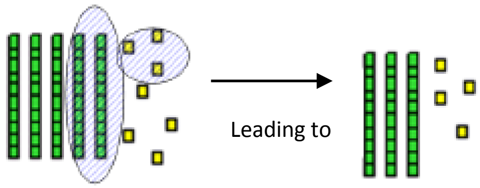
Bridging through 1.0

Encourage efficient partitioning.

1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning
5.7 – 2.3 = 3.4
Count out 5.7. Remove 2.3.



Subtraction- Year 5

In Year 5, decimal subtraction should then be transferred across into compact decomposition method in the same way it was with whole numbers in Year 4.
By the end of Year 5, all children must be secure in abstract written column method up to six digits or 2 decimal places without use of manipulatives.

Gradation of difficulty- subtraction:

Lower School

1. No exchange
e.g. 9 – 2 = 7
27 – 14 = 13

$$\begin{array}{rcc} & T & U \\ & 2 & 7 \\ - & 1 & 4 \\ \hline & 1 & 3 \\ \hline \end{array}$$

2. Exchanging tens for ones
e.g. 165 – 59 = 106
36 – 28 = 8

$$\begin{array}{rcc} & T & U \\ & {}^{2}\cancel{3} & {}^{1}6 \\ - & 2 & 8 \\ \hline & 0 & 8 \\ \hline \end{array}$$

To complete 6 – 8 in units column, we have to exchange one from the tens.
Units –> 6 becomes 16
Tens –> 3 becomes 2

3. Exchanging hundreds for tens
e.g. 446 – 283 = 163
526 – 373= 153

$$\begin{array}{rccc} & H & T & U \\ & {}^{4}\cancel{5} & {}^{1}\cancel{2} & 6 \\ - & 3 & 7 & 3 \\ \hline & 1 & 5 & 3 \\ \hline \end{array}$$

To complete 2 – 7 in tens column, we have to exchange one from the hundreds.
Tens –> 2 becomes 12
Hundreds –> 5 becomes 4

4. Exchanging hundreds to tens and tens to ones
e.g. 234 – 157 = 77
531 – 375 = 156

$$\begin{array}{rccc} & H & T & U \\ & {}^{4}\cancel{5} & {}^{2}\cancel{3} & 1 \\ - & 3 & 7 & 5 \\ \hline & 1 & 5 & 6 \\ \hline \end{array}$$

To complete 1 – 5 in units column, we have to exchange one from the tens.
Units–> 1 becomes 11
Tens –> 3 becomes 2
AND
To complete 2 – 7 in tens column, we have to exchange one from the hundreds.
Tens –> 2 becomes 12
Hundreds –> 5 becomes 4

5. Exchanging hundreds to tens and tens to ones but with different number of digits
e.g. 114 – 89 = 25
345 – 78 = 267

$$\begin{array}{rccc} & H & T & U \\ & {}^{2}\cancel{3} & {}^{13}\cancel{4} & {}^{1}5 \\ - & & 7 & 8 \\ \hline & 2 & 6 & 7 \\ \hline \end{array}$$

As with step 4 but numbers need to be placed in correct place value columns before calculating.

7. Decimals up to 2 decimal places (same number of decimal places)
e.g. 1.25 – 0.74 = 0.51
23.4 – 11.3 = 12.1

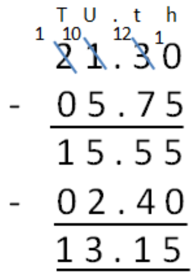
$$\begin{array}{rcccc} & T & U & . & t \\ & 2 & 3 & . & 4 \\ - & 1 & 1 & . & 3 \\ \hline & 1 & 2 & . & 1 \\ \hline \end{array}$$

Gradation of difficulty- subtraction:

Upper School

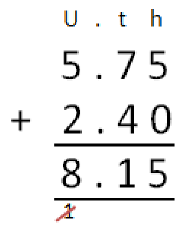
8. Subtract two or more decimals with a range of decimal places
e.g. 12.83 – 8.6 – 2.33 =
21.3 – 5.75 – 2.4 =

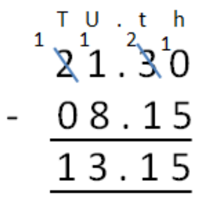
Either subtract and subtract again:



Gaps will be filled with a 0 place holder.

Or, add both subtrahends and subtract in one step:



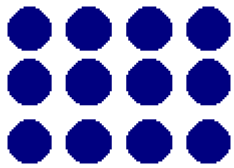


Herts for Learning

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12,

4 x 3 =12,

3 + 3 + 3 + 3 = 12,

4 + 4 + 4 =12.

And it is also a model for division

12 ÷ 4 = 3

12 ÷ 3 = 4

12 – 4 – 4 – 4 = 0

12 – 3 – 3 – 3 – 3 = 0

Multiplication- EYFS/Year 1

Early experiences

Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.

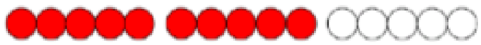
Multiplication- Year 1+

Repeated addition (repeated aggregation)

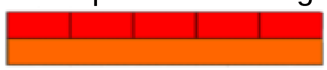
3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

Children learn that repeated addition can be shown on a number line.

Children learn that repeated addition can be shown on a bead string.



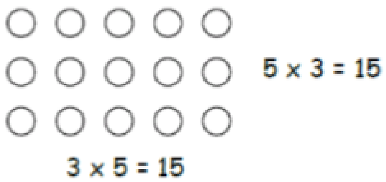
Children also learn to partition totals into equal trains using Cuisenaire Rods



5 x 3 = 15

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



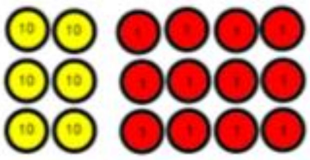
Multiplication- Year 3

Arrays leading into the grid method

Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication.

Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc.

24 x 3 =

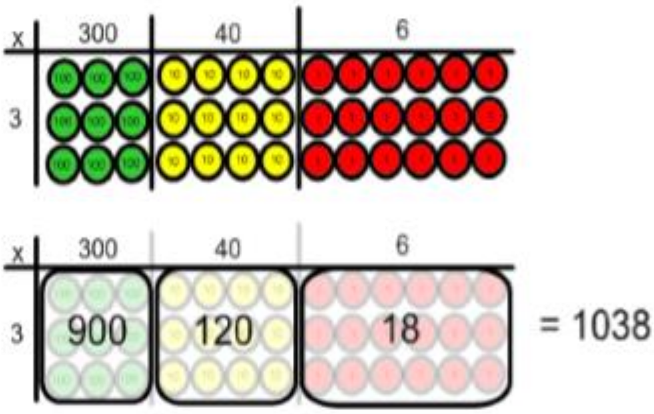


60 + 12

Multiplication- Year 4

Grid method

This written strategy is introduced for the multiplication of TU x U to begin with. It may require column addition methods to calculate the total.



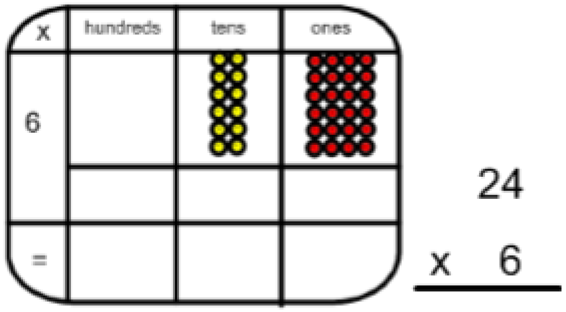
Children will not be expected to draw this diagram out in their books but to record the abstract grid form (as seen in the second picture).

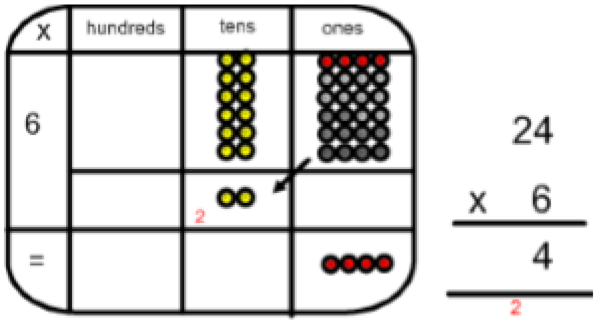
If required, grid method may be used to scaffold this further.

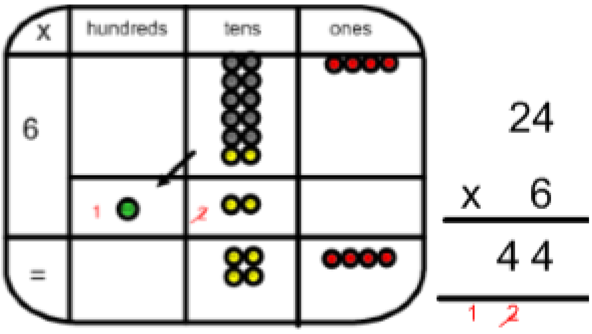
Short multiplication — multiplying by a single digit

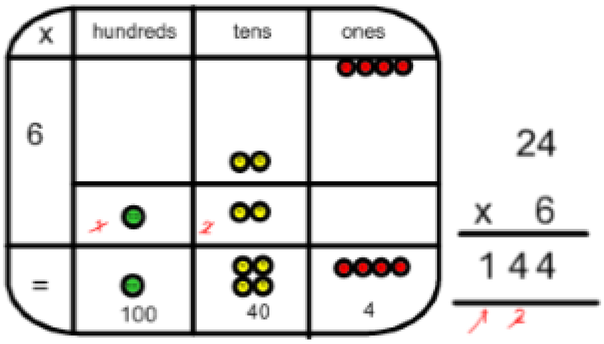
The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging. This is very similar to the previous step- a place value grid and column recording alongside it have been added.

24 x 6


x
hundreds
tens
ones
6
=
24
x 6


x
hundreds
tens
ones
6
2
=
24
x 6
4
2


x
hundreds
tens
ones
6
1
2
=
24
x 6
44
1 2


x
hundreds
tens
ones
6
1
2
=
100
40
4
24
x 6
144
1 2

Multiplication- Year 5
Children will progress to completing HTU x TU calculations involving exchanging using column method:

	Th	H	T	U
		4	8	2
x			1	5
	2	4	1	0
	$\cancel{2}$	$\cancel{4}$	$\cancel{1}$	
+	4	8	2	0
	7	2	3	0
	$\cancel{1}$			

Multiplication- Year 6

Children will progress to completing ThHTU x TU calculations involving exchanging using column method below:

$$
\begin{array}{cccccc}
 & \text{TTh} & \text{Th} & \text{H} & \text{T} & \text{U} \\
 & & 2 & 6 & 3 & 8 \\
\times & & & & 2 & 4 \\
\hline
 & 1 & 0 & 5 & 5 & 2 \\
 & \cancel{1} & \cancel{2} & \cancel{1} & \cancel{3} & \\
+ & 5 & 2 & 7 & 6 & 0 \\
 & \cancel{1} & & \cancel{1} & & \\
\hline
 & 6 & 3 & 3 & 1 & 2 \\
\hline
 & & \cancel{1} & \cancel{1} & &
\end{array}
$$

Decimal multiplication

e.g. 1.2 x 0.34

1) Children will count the amount of digits after the decimal place in the calculation. They will save this number for later

1.2 x 0.34   1.2 has 1dp and 0.34 has 2dp which totals 3dps

2) Remove the decimal point from the numbers

12 x 034 = 12 x 34

3) Line up in column method as whole numbers

$$
\begin{array}{ccc}
 & 3 & 4 \\
\times & 1 & 2 \\
\hline
\end{array}
$$

4) Calculate

$$
\begin{array}{cccc}
 & & 3 & 4 \\
\times & & 1 & 2 \\
\hline
 & & 6 & 8 \\
+ & 3 & 4 & 0 \\
\hline
 & 4 & 0 & 8 \\
\hline
\end{array}
$$

5) Add the total amount of decimal places you started with back into the number, counting in

from the digit furthest to the right

$$\begin{array}{rr} & 34 \\ \times & \underline{12} \\ & 68 \\ + & \underline{340} \\ & .\underline{408} \end{array}$$

Answer = 0.408

Gradation of difficulty- multiplication:

Lower School

1. TU x U no exchange

e.g. 12 x 3 = 36
11 x 5 = 55

$$\begin{array}{rcc} & T & U \\ & 1 & 1 \\ \times & & 5 \\ \hline & 5 & 5 \\ \hline \end{array}$$

2. TU x U with exchange of units to tens

e.g. 25 x 3 = 75
18 x 4 = 72

$$\begin{array}{rcc} & T & U \\ & 1 & 8 \\ \times & & 4 \\ \hline & 7 & 2 \\ \hline & \cancel{3} & \end{array}$$

(Three tens exchanged under)

8 x 4 = 32
The three is exchanged under into the tens column

3. HTU x U no exchange

e.g. 121 x 3 = 363
211 x 4 = 844

$$\begin{array}{rccc} & H & T & U \\ & 2 & 1 & 1 \\ \times & & & 4 \\ \hline & 8 & 4 & 4 \\ \hline \end{array}$$

4. HTU x U with exchange of units to tens

e.g. 126 x 3 = 378
116 x 5 = 580

$$\begin{array}{rccc} & H & T & U \\ & 1 & 1 & 6 \\ \times & & & 5 \\ \hline & 5 & 8 & 0 \\ \hline & & \cancel{3} & \end{array}$$

(Three tens exchanged under)

6 x 5 = 30
The three is exchanged under into the tens column

5. HTU x U with exchange of tens to hundreds

e.g. 132 x 4 = 528
141 x 5 = 705

$$\begin{array}{rccc} & H & T & U \\ & 1 & 4 & 1 \\ \times & & & 5 \\ \hline & 7 & 0 & 5 \\ \hline & \cancel{2} & & \end{array}$$

(Two hundreds exchanged under)

4 in tens x 5 = 20
The two is exchanged under into the hundreds column

Lower School

6. HTU x U with exchange of units to tens and tens to hundreds
e.g. 135 x 5 = 675
126 x 6 = 756

$$\begin{array}{cccc} & H & T & U \\ & 1 & 2 & 6 \\ \times & & & 6 \\ \hline & 7 & 5 & 6 \\ \hline & \cancel{1} & \cancel{3} & \end{array}$$

(Three tens exchanged under)
(One hundred exchanged under)

a) 6 x 6 in units = $\underline{3}6$
The three is exchanged under into the tens column
b) 6 x 2 in tens = $\underline{1}2$
+ exchanged 3 = 15
The one is exchanged under into the hundreds column

7. Continue steps 3-6 but increasing the size of the numbers

Gradation of difficulty- multiplication:

Upper School

8. U.t x U no exchange
e.g. 1.2 x 3 = 3.6
1.1 x 5 = 5.5

$$\begin{array}{ccc} & 1 & 1 \\ \times & & 5 \\ \hline & 5. & 5 \\ \hline \end{array}$$

a) Take out decimal (11 x 5) and count dps
b) Complete calculation
c) Add in dps

9. U.t with exchange of tenths to units
e.g. 2.5 x 3 = 7.5
1.8 x 4 = 7.2

$$\begin{array}{ccc} & 1 & 8 \\ \times & & 4 \\ \hline & 7. & 2 \\ \hline & \cancel{3} & \end{array}$$

(Three tens exchanged under)

a) Take out decimal (18 x 4) and count dps
b) 8 x 4 = $\underline{3}2$ so exchange two under tens column and complete calculation
c) Add in dps

10. As 8-9 but with greater number of digits which may include a range of decimal places x U

Division- EYFS/Year 1

Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.

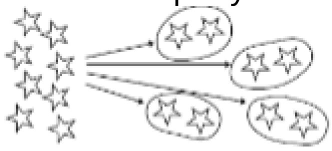
Division- Year 1

Children will be taught the difference between grouping and sharing.

Sharing equally

When sharing, children are taught the ‘one for you, one for me’ method and count how many each person has.

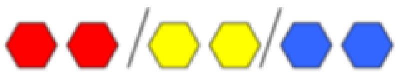
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



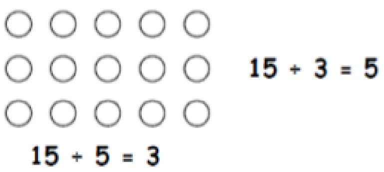
Grouping or repeated subtraction

When grouping, children are taught to collect groups of the given quantity and count how many groups there are.

There are 6 sweets. How many people can have 2 sweets each?



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the ‘bus stop method’ in a written method. This model also connects division to **finding fractions** of discrete quantities.

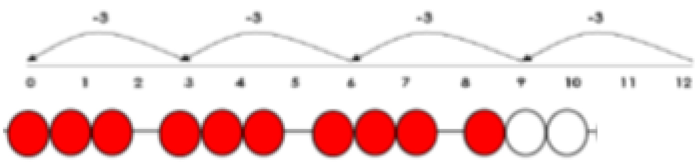


Division- Year 2

Children learn that division is **not** commutative and link this to subtraction.

Repeated subtraction using a bead string or number line

12 ÷ 3 = 4

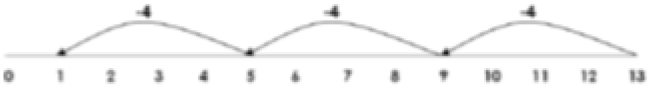


The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as ‘how many 3s make 12?’

Grouping involving remainders

Children move onto calculations involving remainders.

13 ÷ 4 = 3 r1



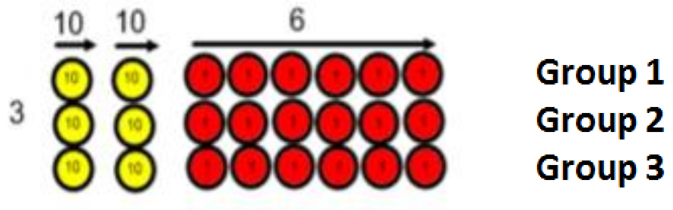
Or using a bead string (see above).

Division- Year 3

Arrays leading into chunking and then long and short division

Children use place value counters and share into rows. This must be completed in rows with the number you are dividing by (divisor) forming the amount of rows on the left (in the same position it would be in for bus stop method).

e.g. 78 ÷ 3



a) Children count out 78 in place value counters
b) Starting with the greatest value column- share the 70 into three equal piles
c) When there are not enough counters left to share equally, exchange them for units
d) Share your units equally into the three rows
e) Count how many are in one row- this is your answer

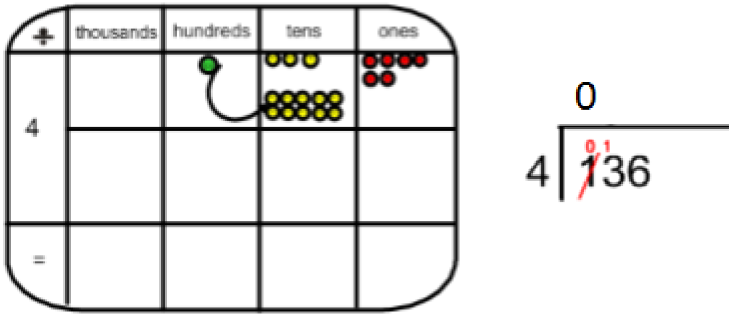
Herts for Learning

Division- Year 4

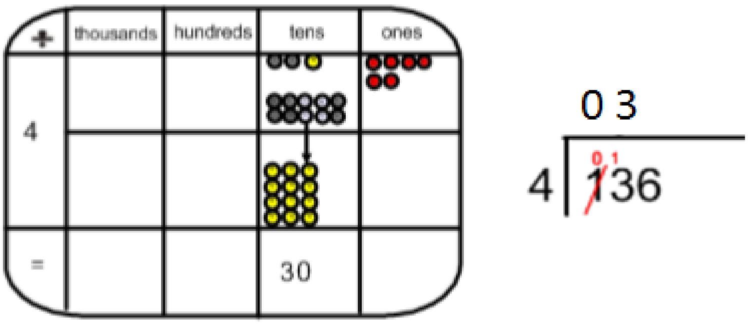
Short division — dividing by a single digit

We can begin to group counters into an array to show short division working

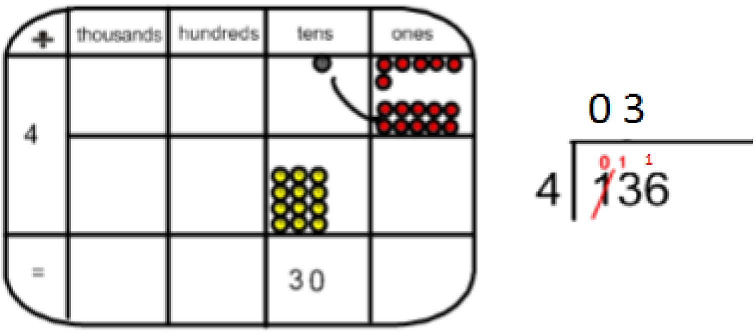
136 ÷ 4



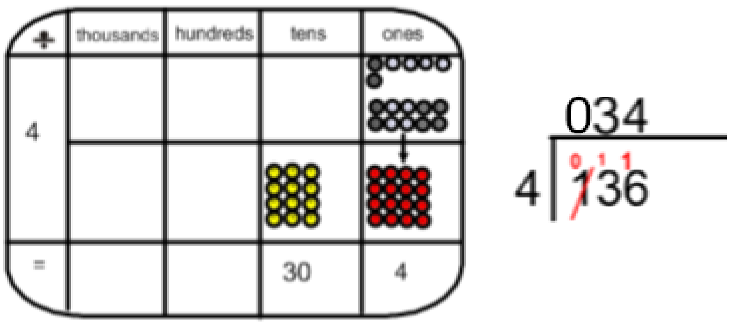
Starting from left:
1 in hundreds column cannot be shared equally into four groups so exchange it for ten 10's counters.
Record a 0 in the hundreds column.



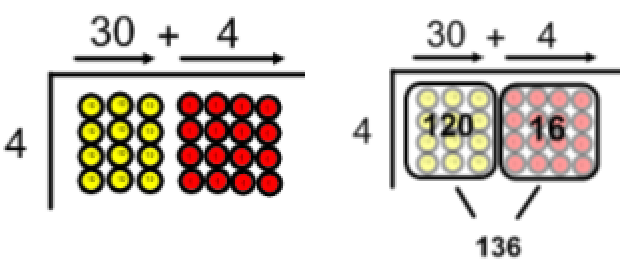
Organise tens counters into four equal groups/rows.
There are 3 in each row- record this in the tens column above the bus stop.



There is one tens counter left which cannot be shared in the tens column- exchange it for ten units.



Share units counters into 4 equal groups/rows.
There are 4 in each group- record this on top of the bus stop in the units column.



Division- Year 5

Children must be able to complete the short method for division without creating arrays or using manipulatives by the end of year 5.

Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.
e.g.:

- I have 62p. How many 8p sweets can I buy?
- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

1. Whole number remainder

$$\begin{array}{l} \quad 13 \;\; \text{r}2 \\ 5\,\overline{)\,6\,{}^{1}7} \end{array}$$

2. Remainder expressed as a fraction of the divisor

$$\begin{array}{l} \quad 13 \;\; \text{r}\,\tfrac{2}{5} \\ 5\,\overline{)\,6\,{}^{1}7} \end{array}$$

3. Remainder expressed as a simplified fraction

4. Remainder expressed as a decimal

$$\begin{array}{l} \quad 13\,.\,4 \\ 5\,\overline{)\,6\,{}^{1}7\,.\,{}^{2}0} \end{array}$$

Gradation of difficulty- short division:

1. TO ÷ O no exchange no remainder
e.g. 28 ÷ 2 = 14

$$\begin{array}{r} 14 \\ 2\overline{\smash{)}28} \end{array}$$

2. TO ÷ O no exchange with remainder
e.g. 67 ÷ 3 = 22 r1

$$\begin{array}{r} 22\ \text{r1} \\ 3\overline{\smash{)}67} \end{array}$$

3. TO ÷ O with exchange no remainder
e.g. 85 ÷ 5 = 17

$$\begin{array}{r} 17 \\ 5\overline{\smash{)}8{}^{3}5} \end{array}$$

4. TO ÷ O with exchange, with remainder
e.g. 83 ÷ 3 = 27 r2

$$\begin{array}{r} 27\ \text{r2} \\ 3\overline{\smash{)}8{}^{2}3} \end{array}$$

5. Zero in the quotient
e.g. 816 ÷ 4 = 204

$$\begin{array}{r} 204 \\ 4\overline{\smash{)}81{}^{1}6} \end{array}$$

6. As 1-5 HTO ÷ O

7. As 1-5 greater number of digits ÷ O

8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3

9. Where the divisor is a two-digit number

Division- Year 6

Long division —dividing by more than one digit

Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as either:-

1. Chunking model of long division using Base 10 equipment
2. Sharing model of long division using place value counters

See the following pages for exemplification of these methods.

The vertical method- 'chunking' leading to long division

To be completed before long division if teaching chunking method.

This can be modelled as an array using place value counters.

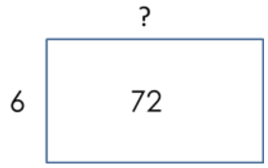
78 ÷ 3 =

$$\begin{array}{rrl} & 78 & \\ - & \underline{30} & (10 \times 3) \\ & 48 & \\ - & \underline{30} & (10 \times 3) \\ & 18 & \\ - & \underline{18} & (6 \times 3) \\ & 0 & \end{array}$$

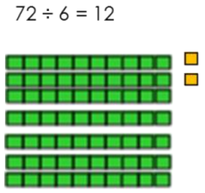
So 78 ÷ 3 = 10 + 10 + 6 =26

Chunking model of long division using Base 10 equipment

This model links strongly to the array representation; so for the calculation 72 ÷ 6 = ? - one side of the array is unknown and by arranging the Base 10 equipment to make the array we can discover this unknown. The written method should be written alongside the equipment so that children make links.

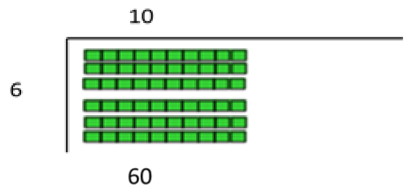


Begin with divisors that are between 5 and 9



6 | 72

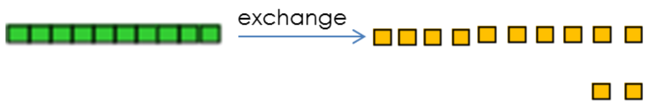
1. Make a rectangle where one side is 6 (the number dividing by) – grouping 6 tens



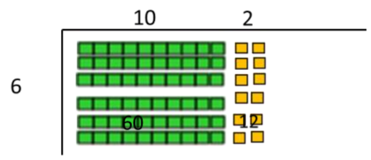
1
6 | 72
- 60 (10 x)
12

After grouping 6 lots of 10 (60) we have 12 left over

2. Exchange the remaining ten for ten ones



3. Complete the rectangle by grouping the remaining ones into groups of 6

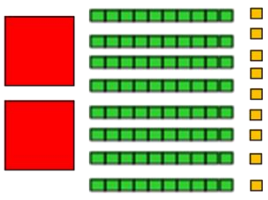


1 2
6 | 72
- 60 (10 x)
12
- 12 (2 x)
0

Move onto working with divisors between 11 and 19

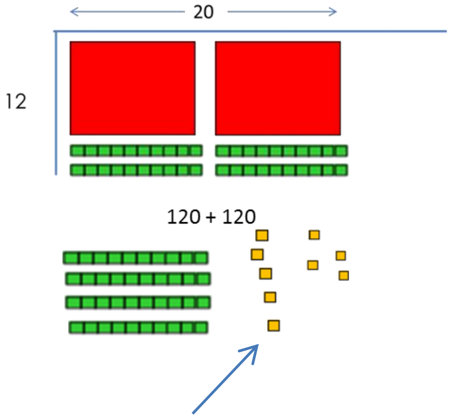
Children may benefit from practise to make multiples of tens using the hundreds and tens and tens and ones.

289 ÷ 12



$$12\overline{)289}$$

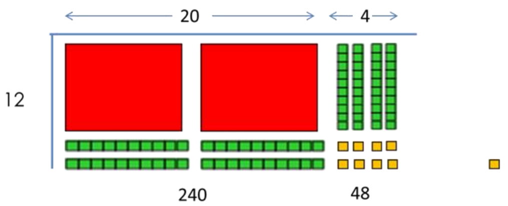
1. Make a rectangle where one side is 12 (the number dividing by) using hundreds and tens



With 49 remaining

$$\begin{array}{rrl} & 2 & \\ 12 & \overline{)289} & \\ - & \underline{240} & (20 \times) \\ & 49 & \end{array}$$

2. Make groups of 12 using tens and ones



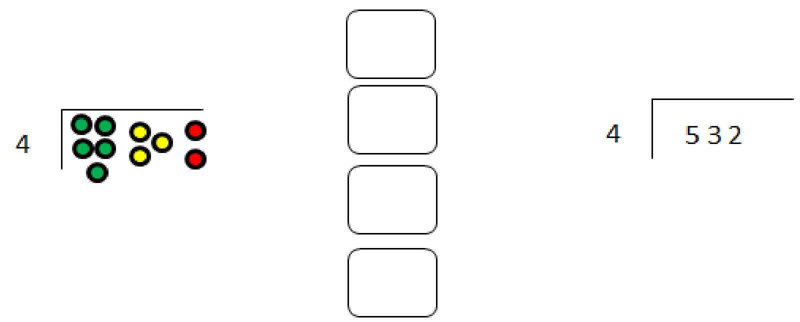
No more groups of 12 can be made and 1 remains

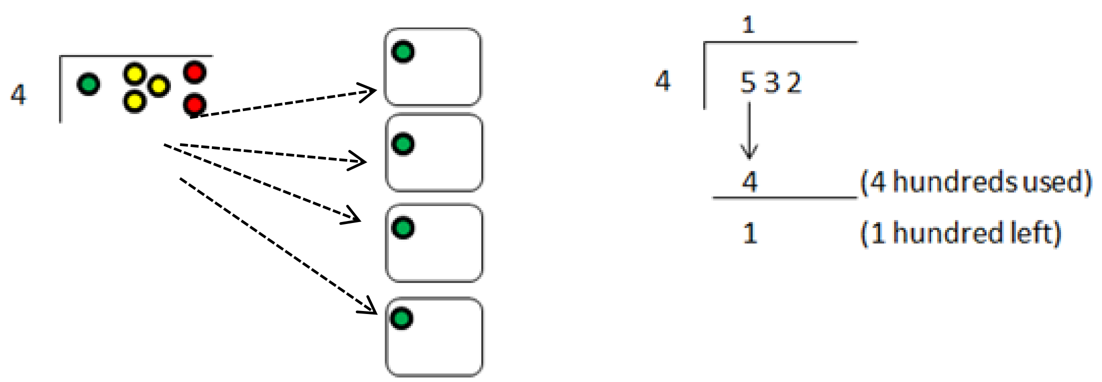
$$\begin{array}{rrl} & 24\ r1 & \\ 12 & \overline{)289} & \\ - & \underline{240} & (20 \times) \\ & 49 & \\ - & \underline{48} & (4 \times) \\ & 1 & \end{array}$$

Sharing model of long division using place value counters

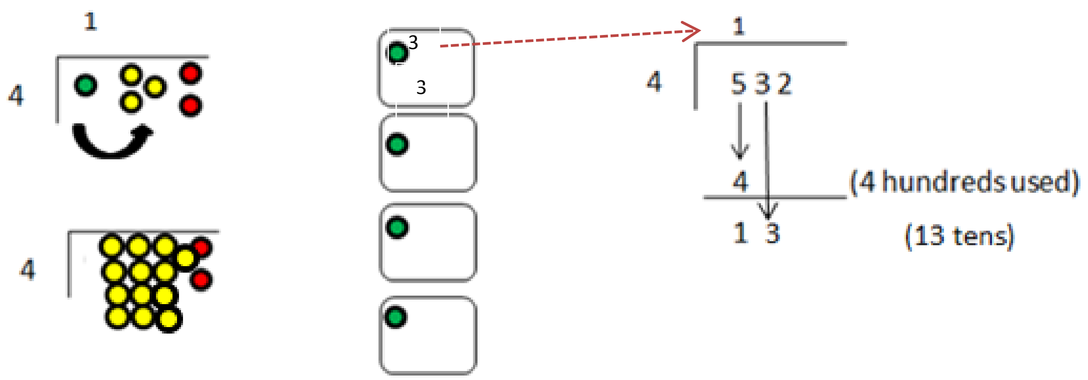
Starting with the most significant digit, share the hundreds. The writing in brackets is for verbal explanation only.

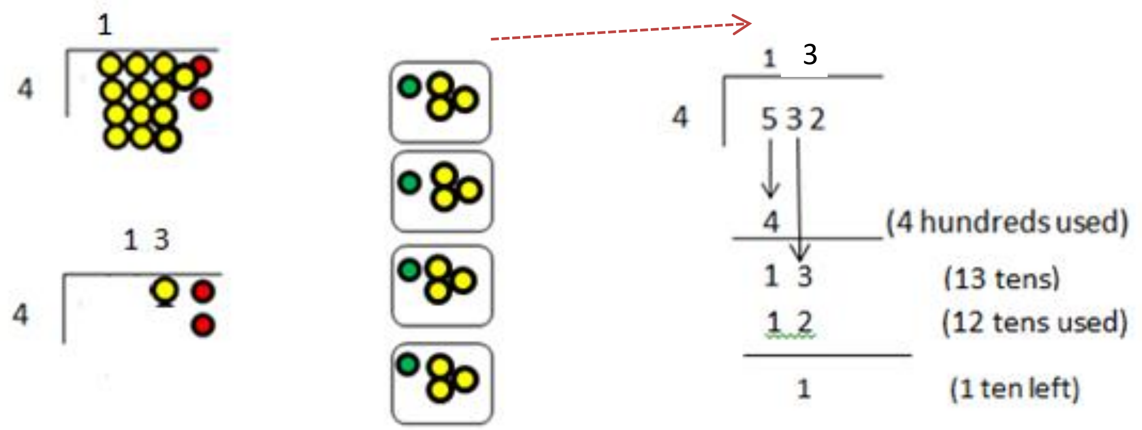
532 ÷ 4



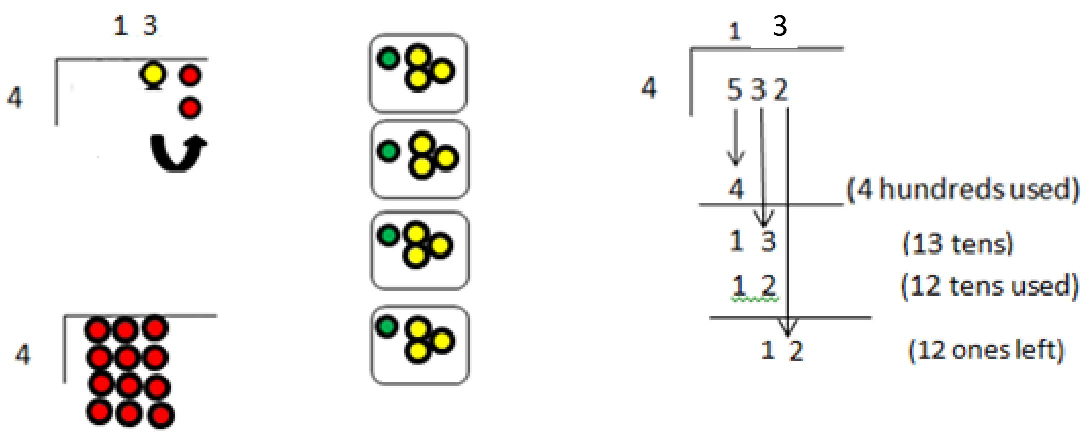


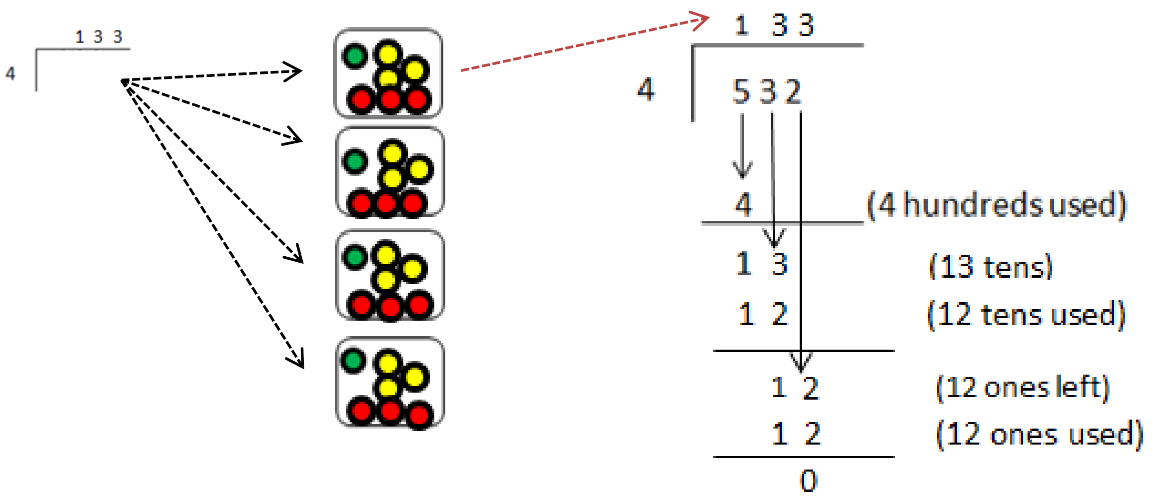
Moving to tens – exchanging hundreds for tens means that we now have a total of 13 tens counters (hence the arrow)





Moving to ones, exchange tens to ones means that we now have a total of 12 ones counters (hence the arrow)



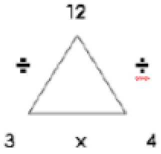


Inverse operations- Year 2

Inverse operations

Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3



Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x Δ = 18   O x □ = 32
24 ÷ 2 = □   15 ÷ O = 3   Δ ÷ 10 = 8

This can also be supported using arrays: e.g.
3 X ? = 12